雷射光學在海洋浮游動物研究之應用

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大綱

- 電磁波簡介
- 雷射原理及光學特性
- 傅氏光學原理
- 傅氏光學在浮游動物研究之應用

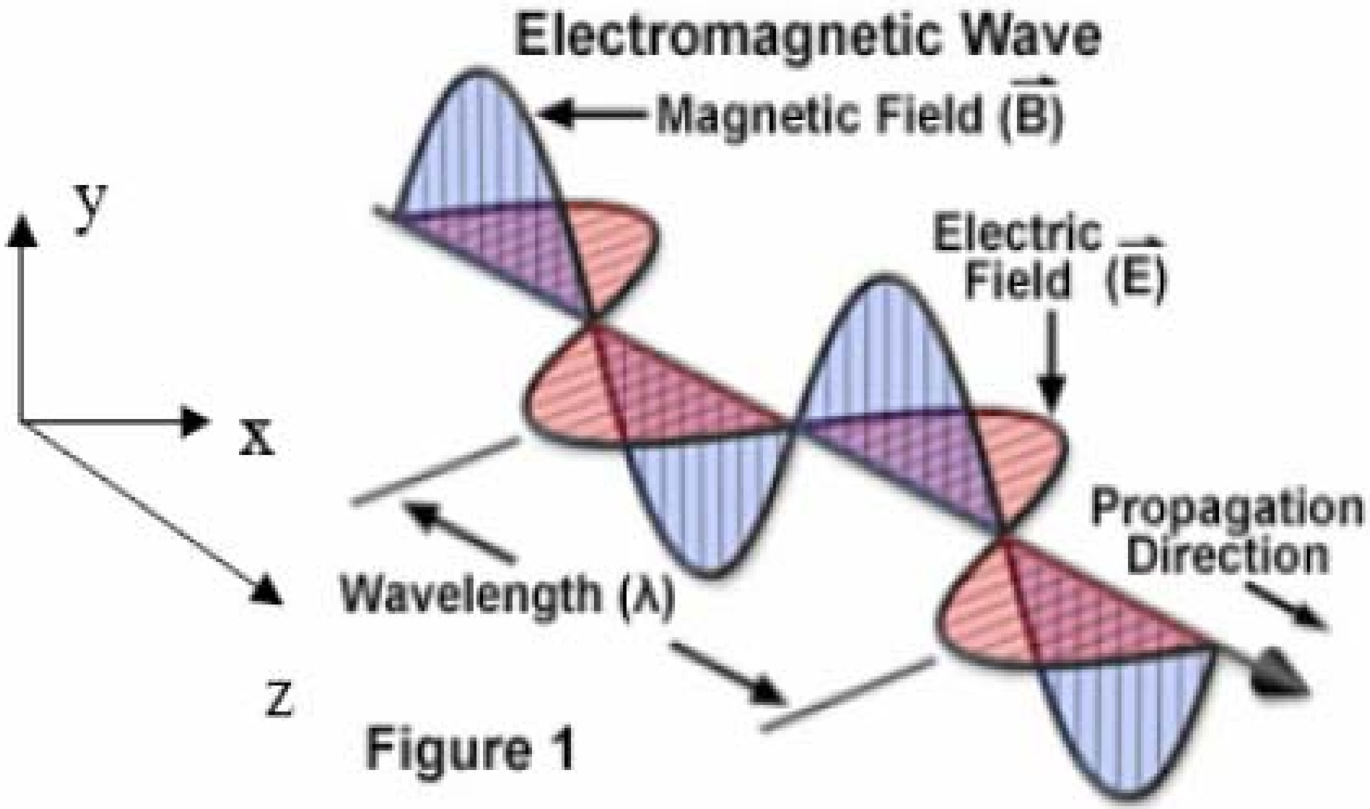


Figure 1

$$\vec{E}(\vec{x},t) = \vec{E}_0 e^{i(\vec{k}\cdot\vec{x}-\omega t)}$$

$$\vec{B}(\vec{x},t) = \vec{B}_0 e^{i(\vec{k}\cdot\vec{x}-\omega t)}$$

$$\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t}$$ (Faraday's law)

$$\nabla \times \mathbf{B} = \mu_0 \mathbf{J} + \mu_0 \varepsilon_0 \frac{\partial \mathbf{E}}{\partial t}$$ (Ampère's law with Maxwell's correction)

$$\nabla \times \left(\nabla \times \mathbf{E}\right) = \nabla\left(\nabla . \mathbf{E}\right) - \nabla^2 \mathbf{E} = \nabla \times \left(-\frac{\partial \mathbf{B}}{\partial t}\right)$$

$$= -\frac{\partial}{\partial t}\left(\nabla \times \mathbf{B}\right) = -\mu_0 \varepsilon_0 \frac{\partial^2 \mathbf{E}}{\partial t^2}$$

$$\nabla \times \left(\nabla \times \mathbf{B}\right) = \nabla\left(\nabla . \mathbf{B}\right) - \nabla^2 \mathbf{B} = \nabla \times \left(\mu_0 \varepsilon_0 \frac{\partial \mathbf{E}}{\partial t}\right)$$

$$= -\mu_0 \varepsilon_0 \frac{\partial}{\partial t}\left(\nabla \times \mathbf{E}\right) = -\mu_0 \varepsilon_0 \frac{\partial^2 \mathbf{B}}{\partial t^2}$$

$$\nabla^2 \mathbf{E} = \varepsilon_0 \mu_0 \frac{\partial^2 \mathbf{E}}{\partial t^2}$$

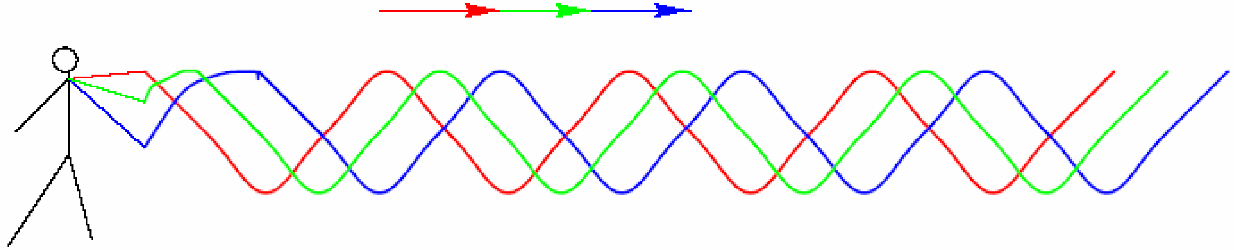
$$\nabla^2 \mathbf{B} = \varepsilon_0 \mu_0 \frac{\partial^2 \mathbf{B}}{\partial t^2}$$

$$\mathbf{S} = c^2 \varepsilon_0 \mathbf{E} \times \mathbf{B}$$

$$c = \frac{1}{\sqrt{(\varepsilon_0 \mu_0)}}, \quad v = \frac{1}{\sqrt{(\varepsilon \mu)}}, \quad n = \frac{c}{v} = \sqrt{(\frac{\varepsilon \mu}{\varepsilon_0 \mu_0})}$$

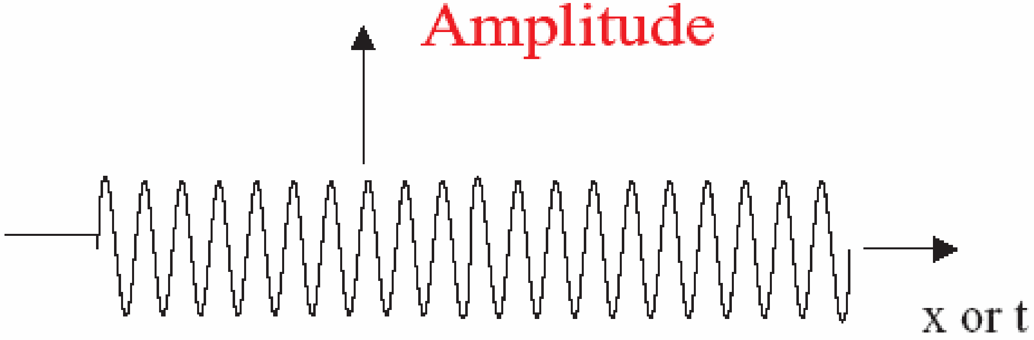
$$n \approx \sqrt{\frac{\varepsilon}{\varepsilon_0}}$$

$$2.99792458 \times 10^8 \text{ ms}^{-1}$$

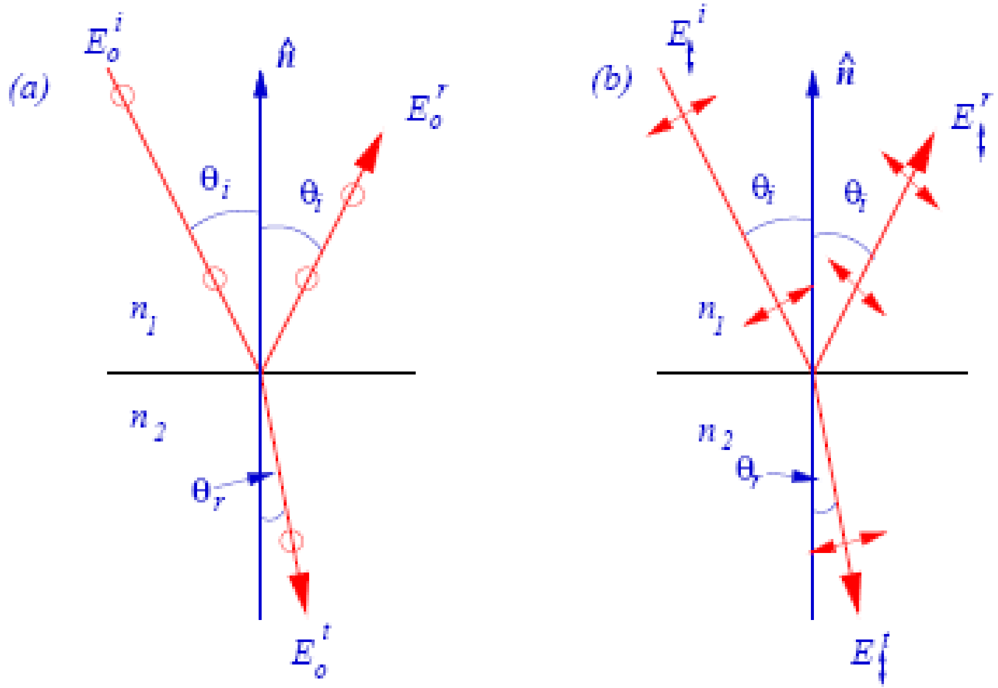


$$y = A\cos(2\pi \tfrac{x}{\lambda} - 2\pi \tfrac{t}{T}) \text{ or } y = A\cos(kx - \omega t)$$

$$y = Ae^{i(kx-\omega t)}$$

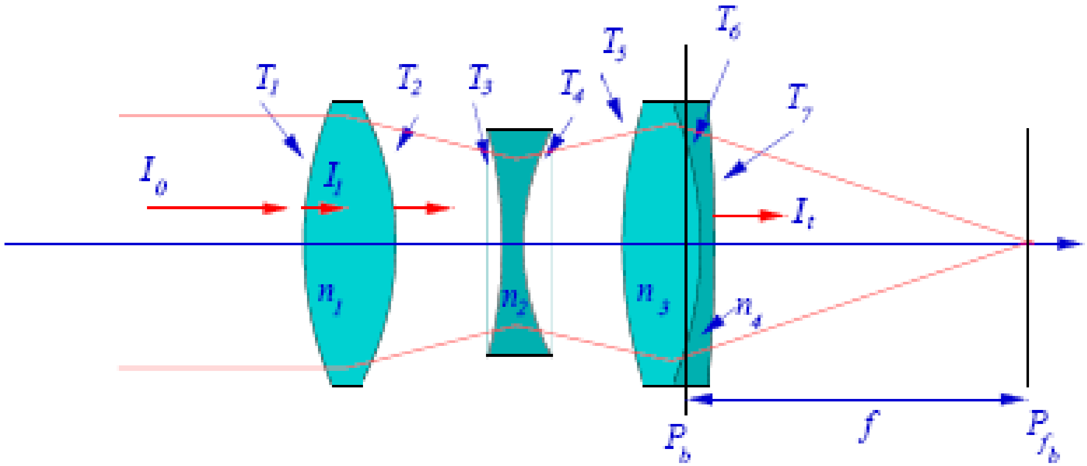


Reflection at Boundaries



Snell's Law $n_1 \sin\theta_1 = n_2 \sin\theta_2$

Multi-element lens,



For incident I_0, after the first boundary we get

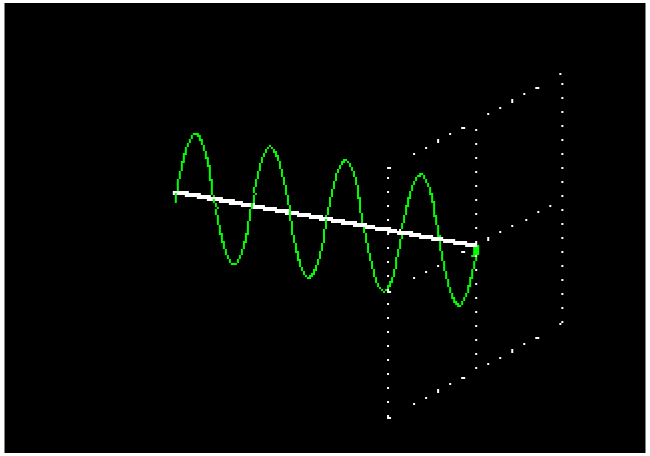
$$I_1 = T_1 I_o$$

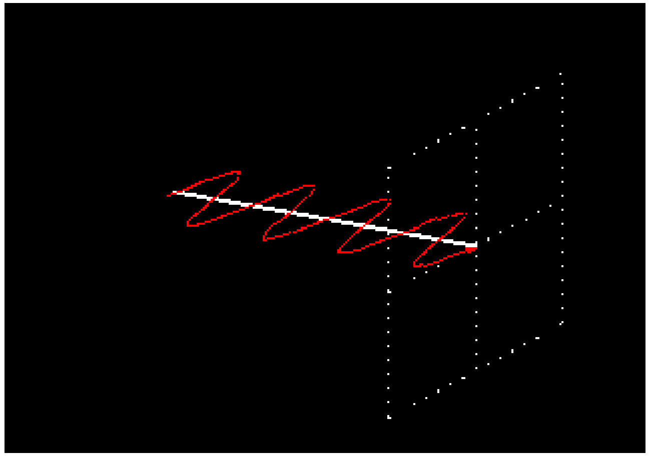
so in this case after 7 boundaries,

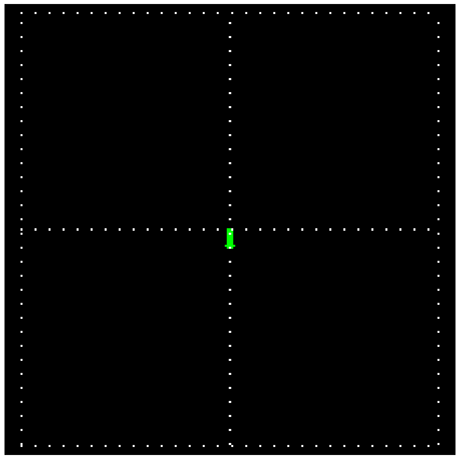
$$I_t = T_1 T_2 \cdots T_7 I_0 = \prod_{i=1}^{N} T_i I_0$$

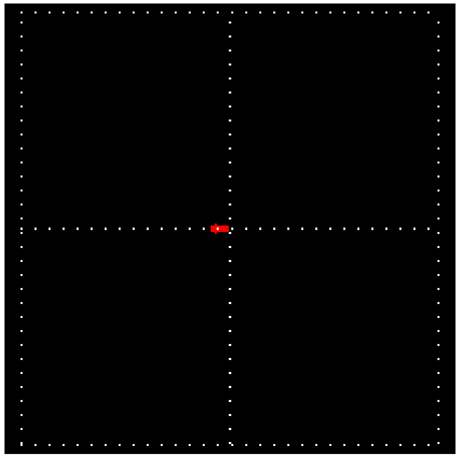
線偏極化
Linear-polarized waves

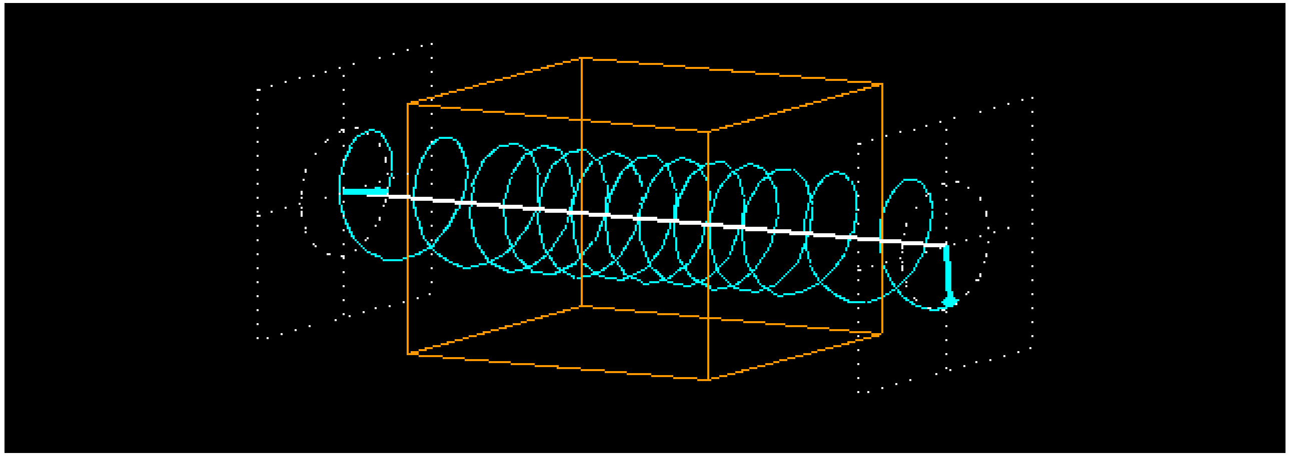
$$\Psi(z,t) = \mathbf{E}\cos(\kappa z - \omega t)$$

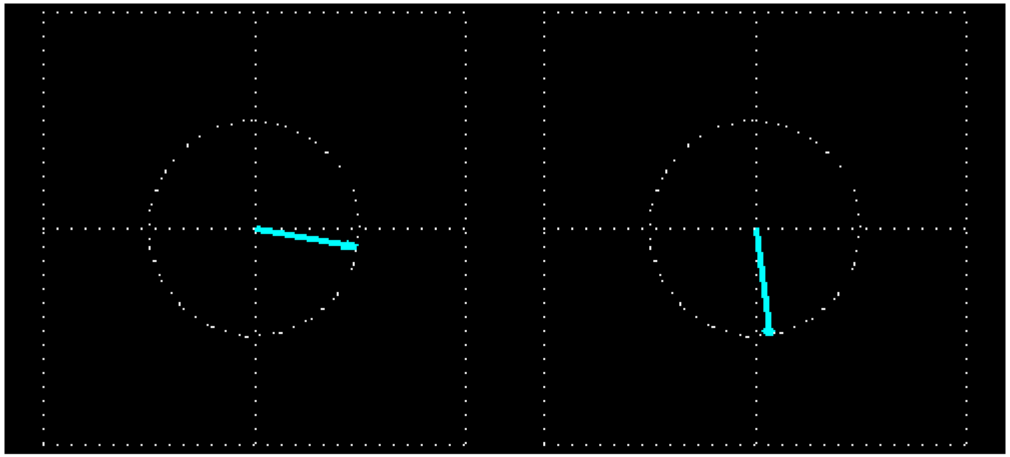












The Fourier Series

$$f(x) = \frac{1}{2}a_0 + \sum_{n=1}^{\infty} a_n \cos(nx) + \sum_{n=1}^{\infty} b_n \sin(nx)$$

$$a_0 = \frac{1}{\pi}\int_{-\pi}^{\pi} f(x)dx$$

$$a_n = \frac{1}{\pi}\int_{-\pi}^{\pi} f(x)\cos(nx)dx$$

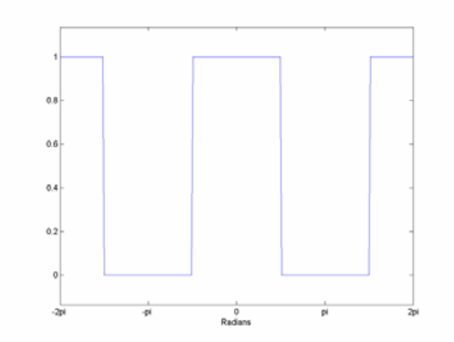
$$b_n = \frac{1}{\pi}\int_{-\pi}^{\pi} f(x)\sin(nx)dx$$

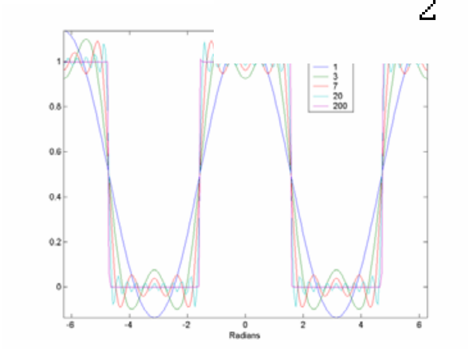
$$a_0 = \frac{1}{\pi}\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} 1dx = \frac{1}{\pi}[x]_{-\frac{\pi}{2}}^{\frac{\pi}{2}} = 1$$

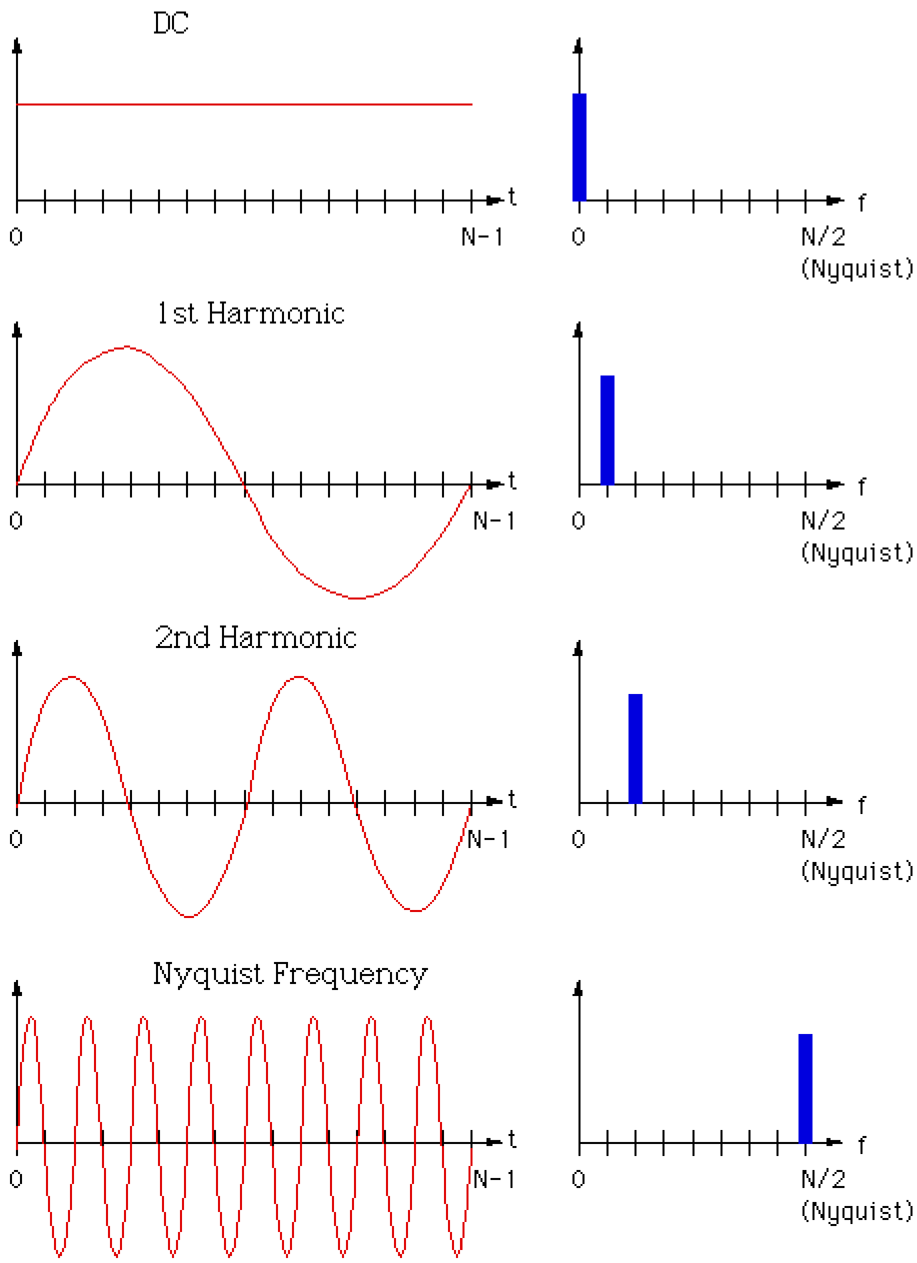
$$a_n = \frac{2}{\pi n}\sin\left(\frac{\pi n}{2}\right)$$

$$b_n = 0$$

$$f(x) = \frac{1}{2} + \sum_{n=1,3,5,\ldots}^{\infty} \frac{2}{\pi n}\cos(nx)$$







DC
t
0
N-1
f
0
N/2
(Nyquist)
1st Harmonic
t
0
N-1
f
0
N/2
(Nyquist)
2nd Harmonic
t
0
N-1
f
0
N/2
(Nyquist)
Nyquist Frequency
t
0
N-1
f
0
N/2
(Nyquist)

The Complex Fourier Series

$$\sin(nx) = \frac{e^{inx} - e^{-inx}}{2i}$$

$$\cos(nx) = \frac{e^{inx} + e^{-inx}}{2}$$

$$f(x) = \sum_{n=-\infty}^{\infty} c_n e^{inx}$$

$$f(x) = \sum_{-\infty}^{\infty} c_n e^{\frac{i\pi nx}{L}}$$

$$c_n = \frac{1}{2L} \int_{-L}^{L} f(x) e^{\frac{-i\pi nx}{L}} dx$$

The Fourier Transform

$$\mathcal{F}\{f(x)\} = F(u) = \int_{-\infty}^{\infty} f(x) e^{-2\pi i u x} dx$$

$$\mathcal{F}^{-1}\{F(u)\} = f(x) = \int_{-\infty}^{\infty} F(u) e^{2\pi i u x} dx$$

$$\mathcal{F}\{f(x,y)\} = F(u,v) = \iint_{-\infty}^{\infty} f(x,y) e^{-2\pi i (ux+vy)} dx dy$$

$$\mathcal{F}^{-1}\{F(x,y)\} = f(x,y) = \iint_{-\infty}^{\infty} F(u,v) e^{2\pi i (ux+vy)} dx dy$$

Fourier Transform Properties

Scaling

$$\mathcal{F}\{f(ax, by)\} = \frac{1}{|ab|}F\left(\frac{u}{a}, \frac{v}{b}\right)$$

Shifting

$$\mathcal{F}\{f(x-a, y-b)\} = F(u,v)e^{-i2\pi(au+bv)}$$

Convolution

$$g(x,y) = f(x,y) \otimes h(x,y)$$

$$G(u,v) = F(u,v) \cdot H(u,v)$$

$$G(u,v) = \mathcal{F}\left\{\int\int_{-\infty}^{\infty} f(\xi,\eta)h(x-\xi, y-\eta)d\xi d\eta\right\}$$

頻率空間轉換

$$X(\omega) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} x(t) e^{-i\omega t}\, dt$$

$$x(t) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} X(\omega) e^{i\omega t}\, d\omega$$

$$X(\omega) = A(\omega) e^{i\phi(\omega)}$$

$$x(t) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} A(\omega) \cdot e^{i(\omega t + \phi(\omega))}\, d\omega$$

Parseval's Theorem

$$\iint_{\infty}^{\infty} |f(x,y)|^2 \, dx dy = \iint_{-\infty}^{\infty} |F(u,v)|^2 \, du dv$$

Linearity Theorem

$$\mathcal{F}\{af(x,y) + bg(x,y)\} = a\,\mathcal{F}\{f(x,y)\} + b\,\mathcal{F}\{g(x,y)\}$$

Autocorrelation Theorem

$$\mathcal{F}\{f(x,y) \odot f(x,y)\} = \mathcal{F}\left\{\iint_{-\infty}^{\infty} f(\xi,\eta) f^*(\xi - x, \eta - y) d\xi d\eta\right\} = |F(u,v)|^2$$

$$\mathcal{F}\{f(x)\} = F(u) = \int_{-\infty}^{\infty} f(x) e^{-2\pi i u x} dx$$

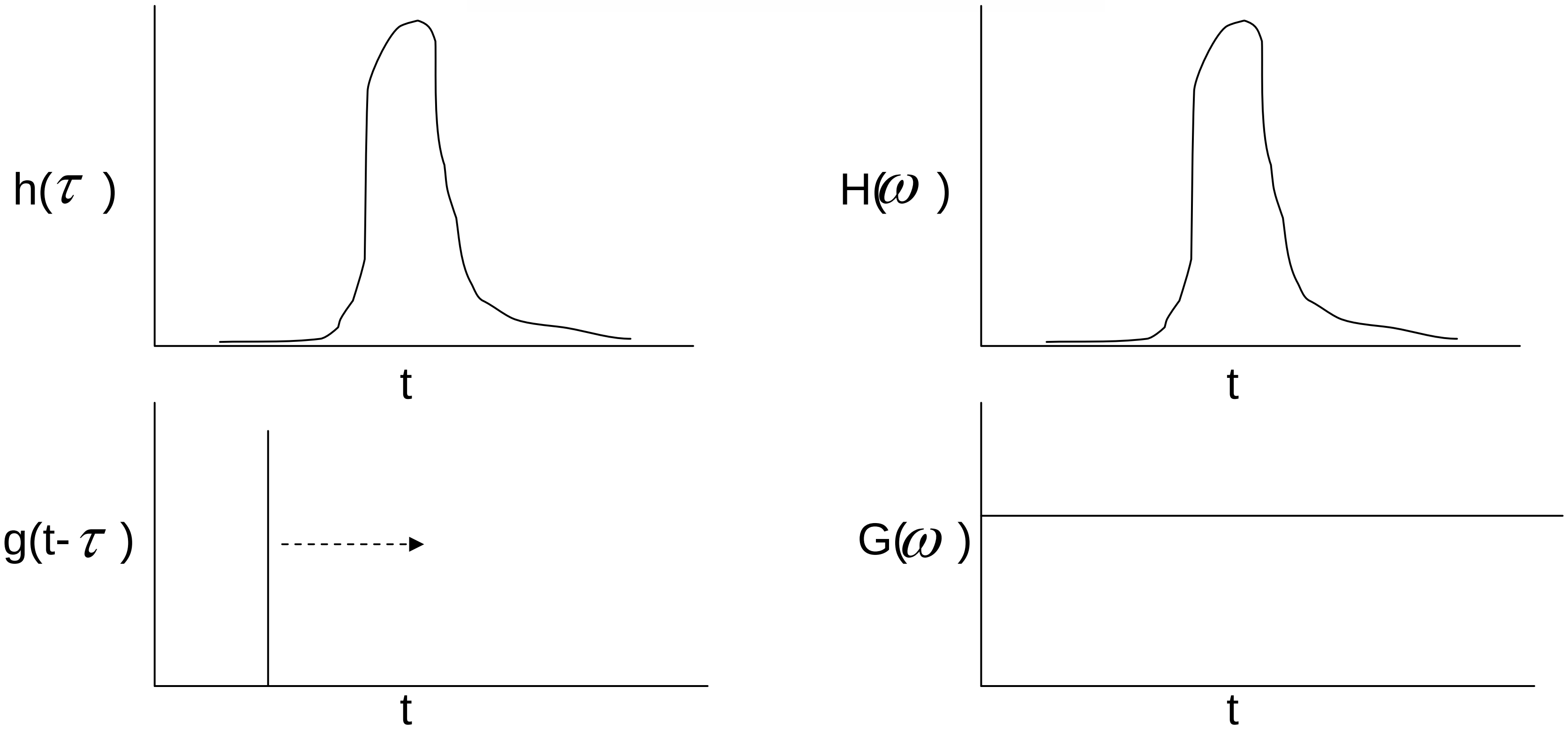
$$\mathcal{F}^{-1}\{F(u)\} = f(x) = \int_{-\infty}^{\infty} F(u) e^{2\pi i u x} dx$$

$$\mathcal{F}\{f(x,y)\} = F(u,v) = \iint_{-\infty}^{\infty} f(x,y) e^{-2\pi i (ux+vy)} dx dy$$

$$\mathcal{F}^{-1}\{F(x,y)\} = f(x,y) = \iint_{-\infty}^{\infty} F(u,v) e^{2\pi i (ux+vy)} dx dy$$

Convolution

$$\int g(t-\tau)h(\tau)d\tau = g \otimes h$$



The Fourier Transform of a Convolution

$$F(u)G(u) = \int_{-\infty}^{\infty} f(x) e^{-2\pi i u x} dx \int_{-\infty}^{\infty} g(x) e^{-2\pi i u x} dx$$

$$F(u)G(u) = \int_{-\infty}^{\infty} f(\tau) e^{-2\pi i u \tau} d\tau \int_{-\infty}^{\infty} g(v) e^{-2\pi i u v} dv$$

$$= \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} e^{-2\pi i u(\tau + v)} f(\tau) g(v) d\tau dv$$

$$x = \tau + v, dx = dv$$

$$F(u)G(u) = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} f(\tau) g(x - \tau) d\tau dx$$

$$= \int_{-\infty}^{\infty} e^{-i2\pi x} \left[\int_{-\infty}^{\infty} f(\tau) g(x - \tau) dtau \right] dx$$

$$= \int_{-\infty}^{\infty} e^{-i2\pi x} f(x) \otimes g(x) dx$$

Commutativity

$$f \otimes g = g \otimes f$$

Scalar multiplication

$$a\left(f \otimes g\right) = (af) \otimes g = f \otimes (ag)$$

where $a \in \mathbb{C}$

Associativity

$$f \otimes \left(g \otimes h\right) = \left(f \otimes g\right) \otimes h$$

Differential rule

If $\mathcal{D}$ is a differential operator

$$\mathcal{D}\left(f \otimes g\right) = \mathcal{D}f \otimes g = f \otimes \mathcal{D}g$$

Distributivity

$$f \otimes (g + h) = \left(f \otimes g\right) + \left(f \otimes h\right)$$

Optical Correlators

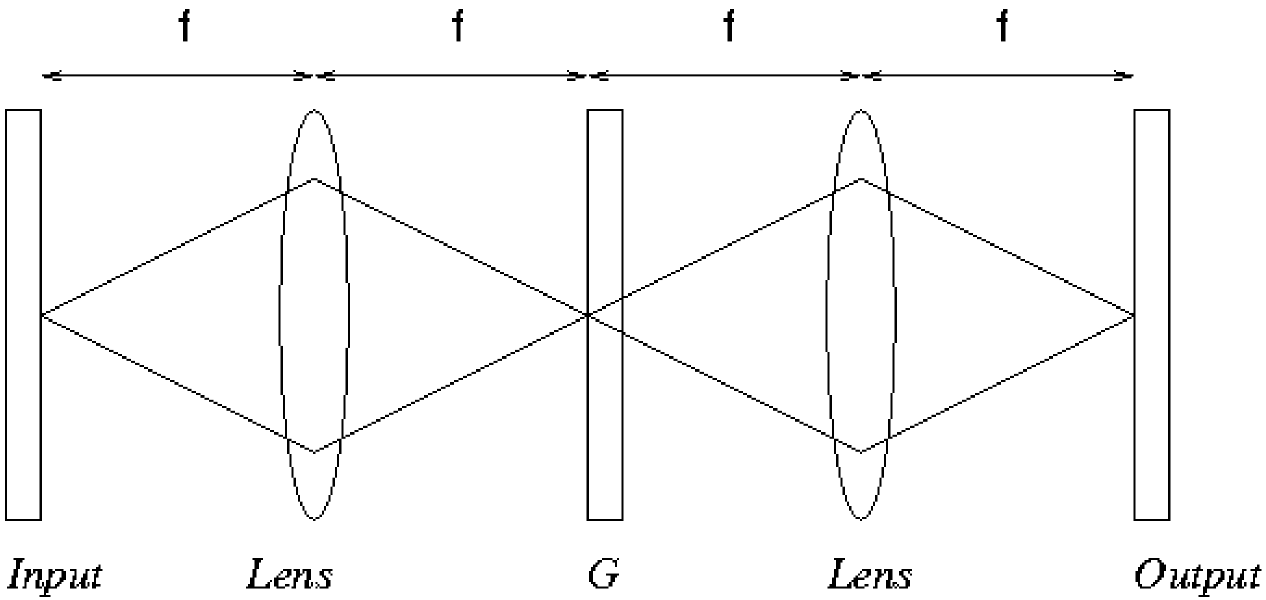
$$c(x) = f(x) \odot g(x) = \int_{-\infty}^{\infty} f(\xi) g(\xi - x) d\xi$$

$$c(x) = f(x) \odot g^*(x) = \int_{-\infty}^{\infty} f(\xi) g^*(\xi - x) d\xi$$

$$f(x) \odot g^*(x) = f(x) \otimes g^*(-x)$$

$$c(x,y) = \mathcal{F}^{-1} \{F(u,v) G^*(-u,-v)\}$$

The 4-f correlator

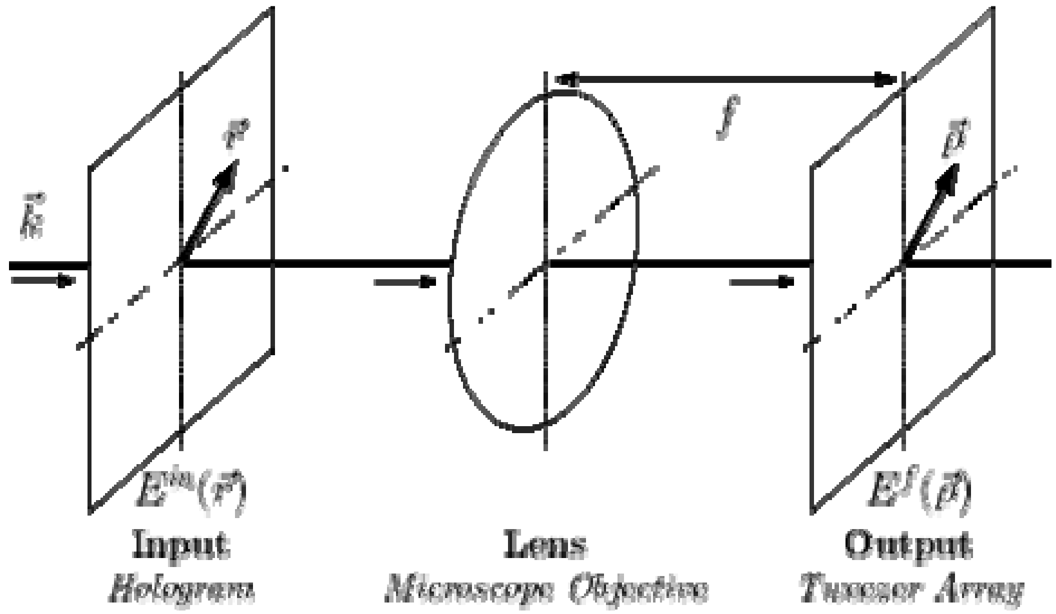


Spatial Invariance

$$F(u)e^{-i2\pi au}G^*(-u)$$

$$C(u) = F(u)G^*(-u)$$

$$C(u)e^{-i2\pi au} \;\therefore\; c(x-a) = f(x-a) \odot g^*(x)$$



$$E^{in}(\vec{r}) = A^{in}(\vec{r})\exp[i\Phi^{in}(\vec{r})], \quad E^{f}(\vec{\rho}) = A^{f}(\vec{\rho})\exp[i\Phi^{f}(\vec{\rho})],$$

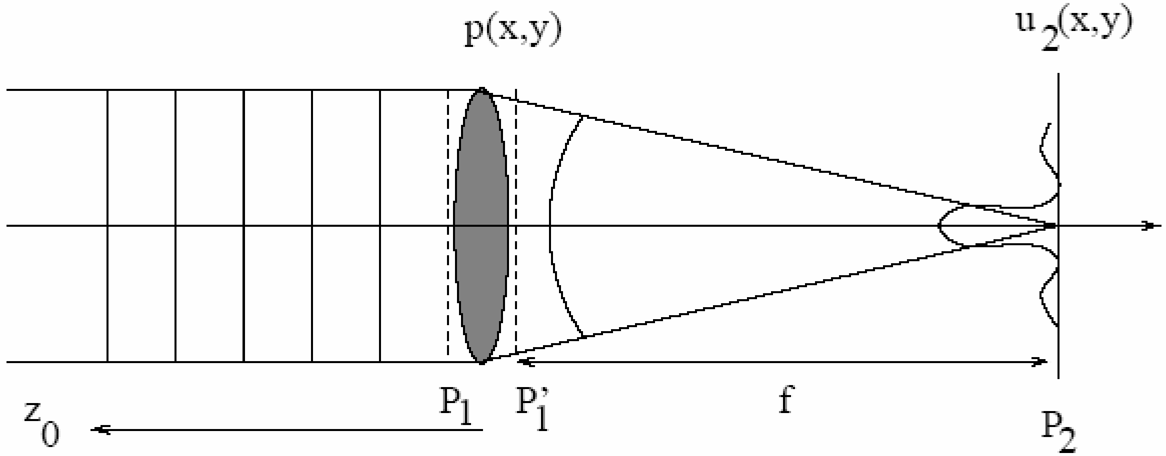
$$I^{f}(\vec{\rho}) = |E^{f}(\vec{\rho})|^{2} = |A^{f}(\vec{\rho})|^{2}$$

$$= \frac{k}{2\pi f} e^{i\theta(\vec{\rho})} \int_{E^f(\vec{\rho})} d^2r \, E^{in}(\vec{r}) \, e^{-ik\vec{r}\cdot\vec{\rho}/f}$$

$$\equiv \mathcal{F}\{E^{in}(\vec{r})\}, \qquad \text{and}$$

$$= \frac{k}{2\pi f} \int_{E^{in}(\vec{r})} d^2\rho \, e^{-i\theta(\vec{\rho})} E^f(\vec{\rho}) \, e^{ik\vec{r}\cdot\vec{\rho}/f}$$

$$\equiv \mathcal{F}^{-1}\{E^f(\vec{\rho})\},$$

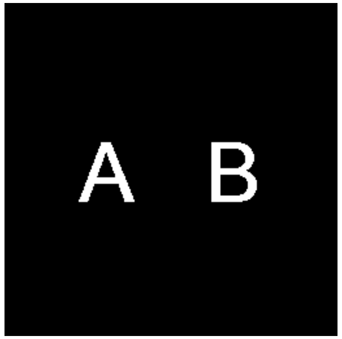


$$u_2(x,y) \quad = \quad B_0 \exp\left(\imath\frac{\kappa}{2f}(x^2+y^2)\right)$$
$$\iint p(s,t)\exp\left(-\imath\frac{\kappa}{f}(sx+ty)\right)\mathrm{d}s\mathrm{d}t$$

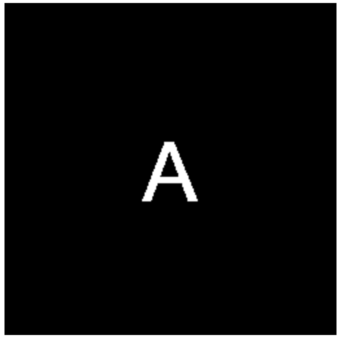
$$P(u,v) = \iint p(x,y)\exp\left(-\imath 2\pi(ux+vy)\right)\mathrm{d}x\mathrm{d}y$$

$$u_2(x,y) = B_0 \exp\left(\imath\frac{\kappa}{2f}(x^2+y^2)\right) P\left(\frac{x}{\lambda f},\frac{y}{\lambda f}\right)$$

Input

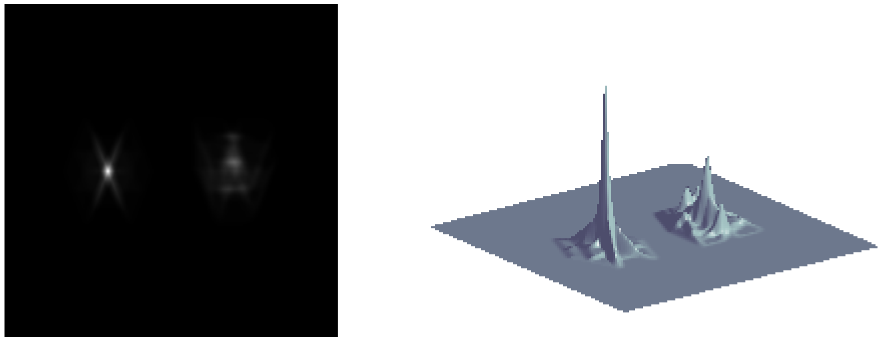


Filter



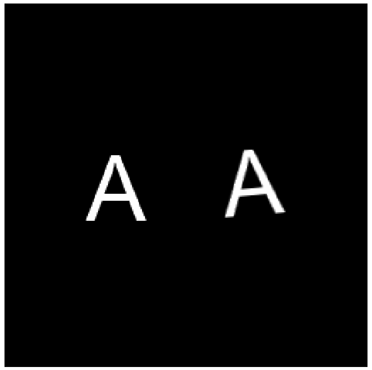
Fourier Transform

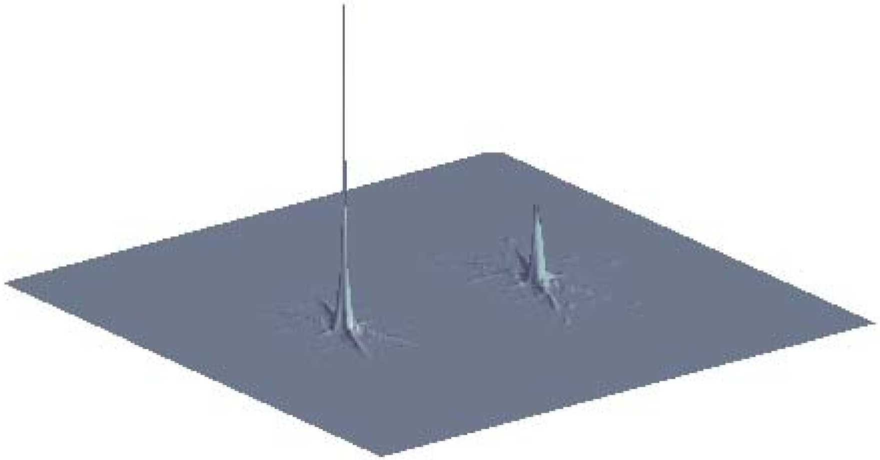
Correlation

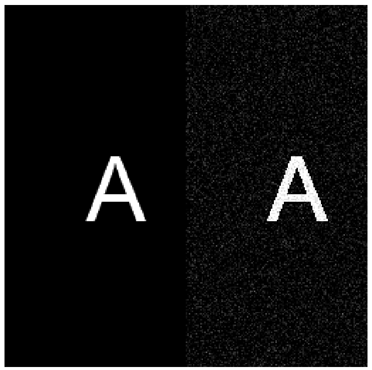


Phase Only Filter

$$F(u,v)\exp(-i\cdot\theta(-u,-v))$$



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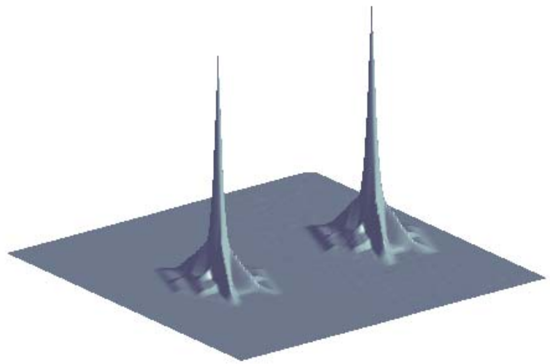
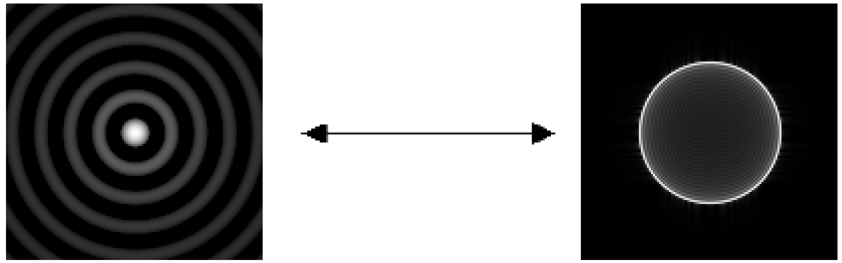
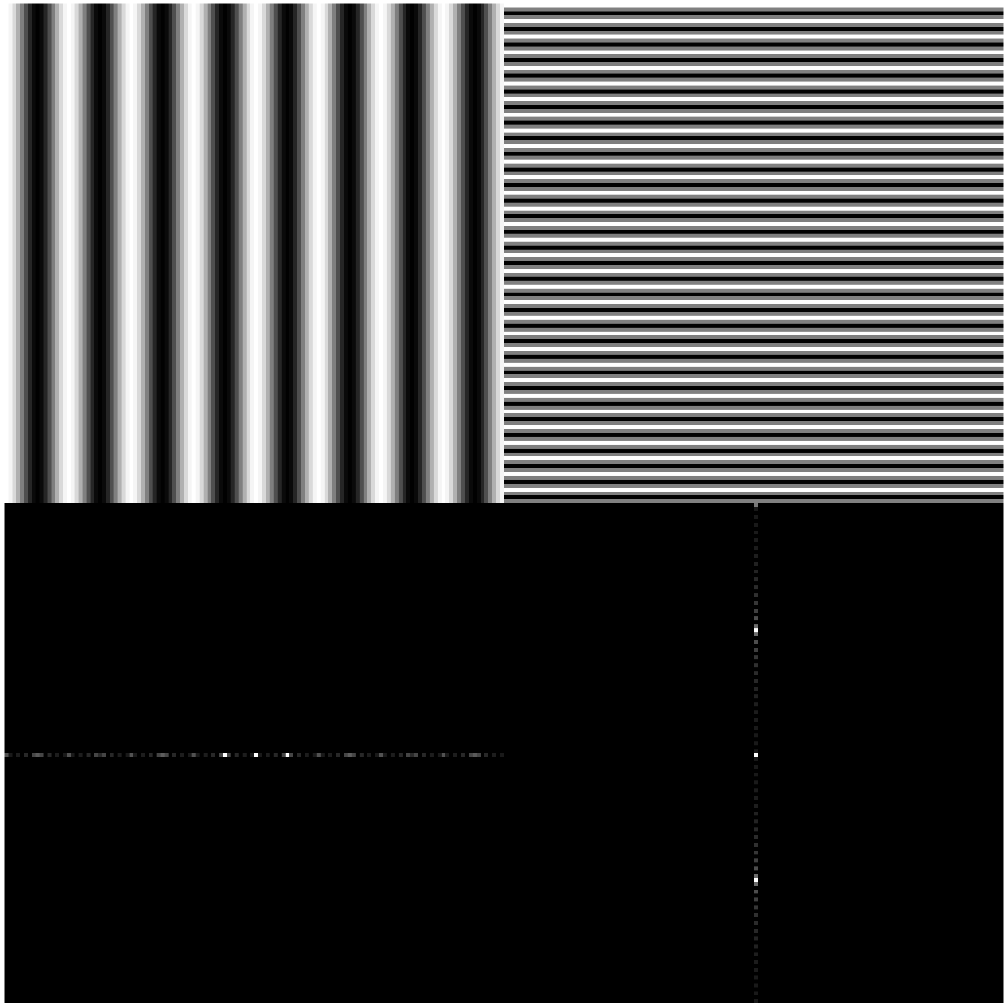
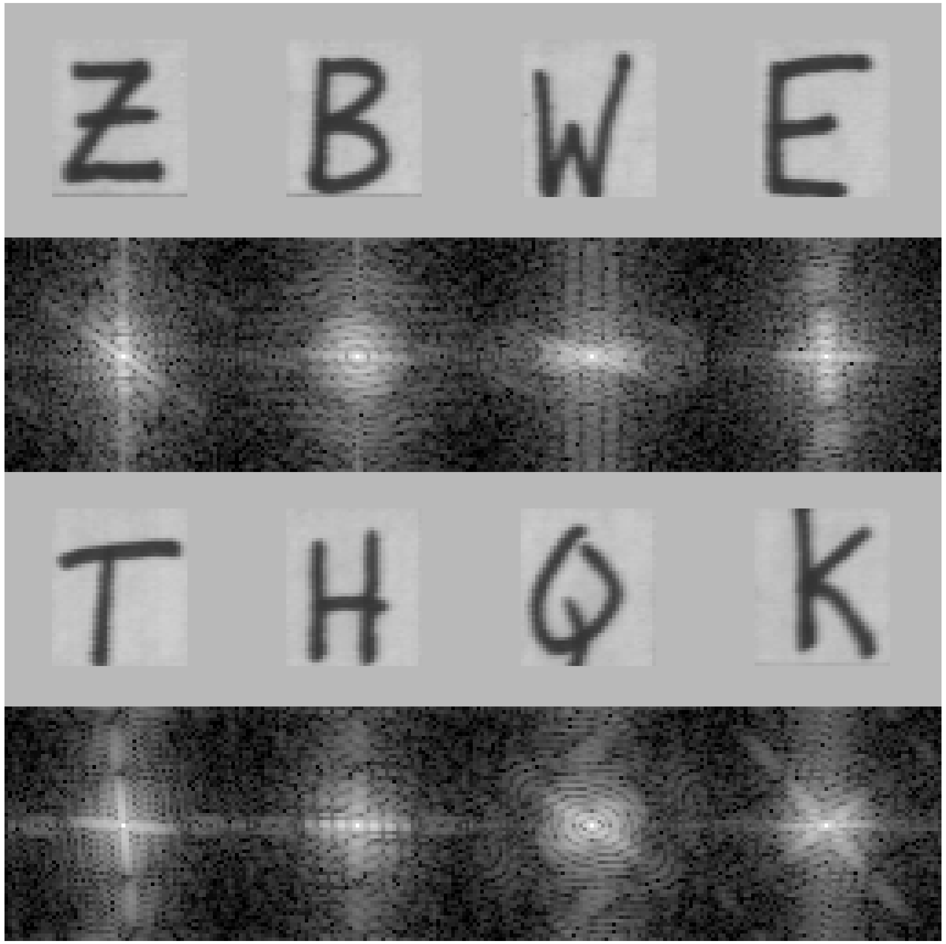
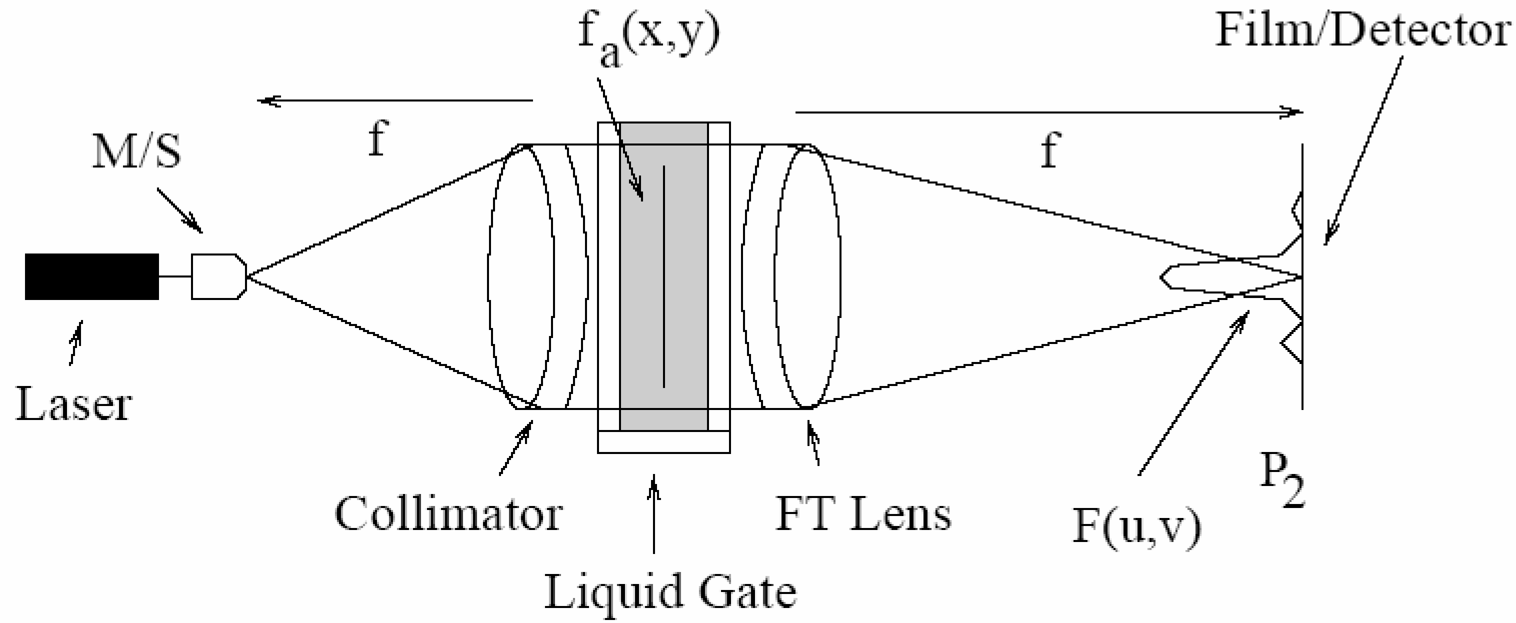
A



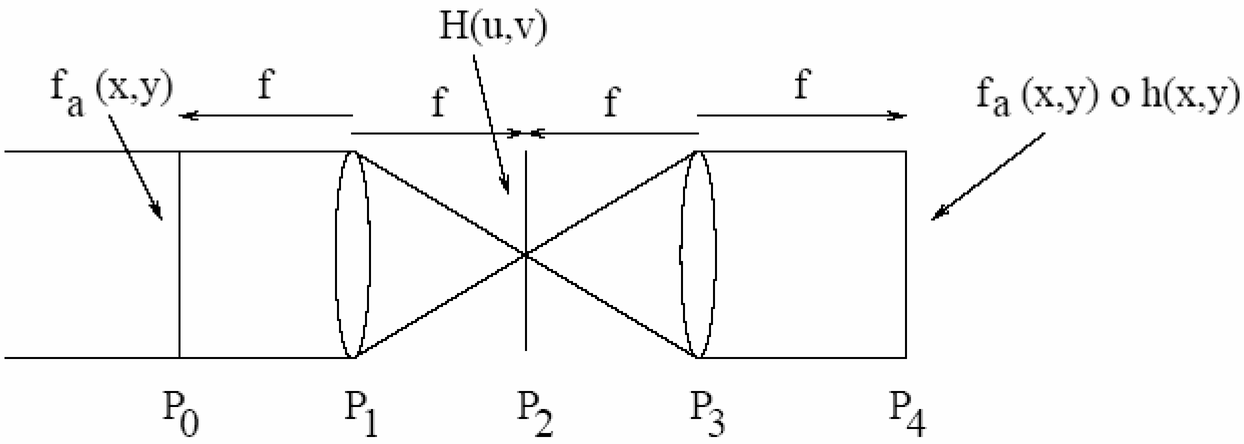








$f_a(x,y)$
Film/Detector
f
M/S
f
Laser
Collimator
Liquid Gate
FT Lens
F(u,v)
P_2



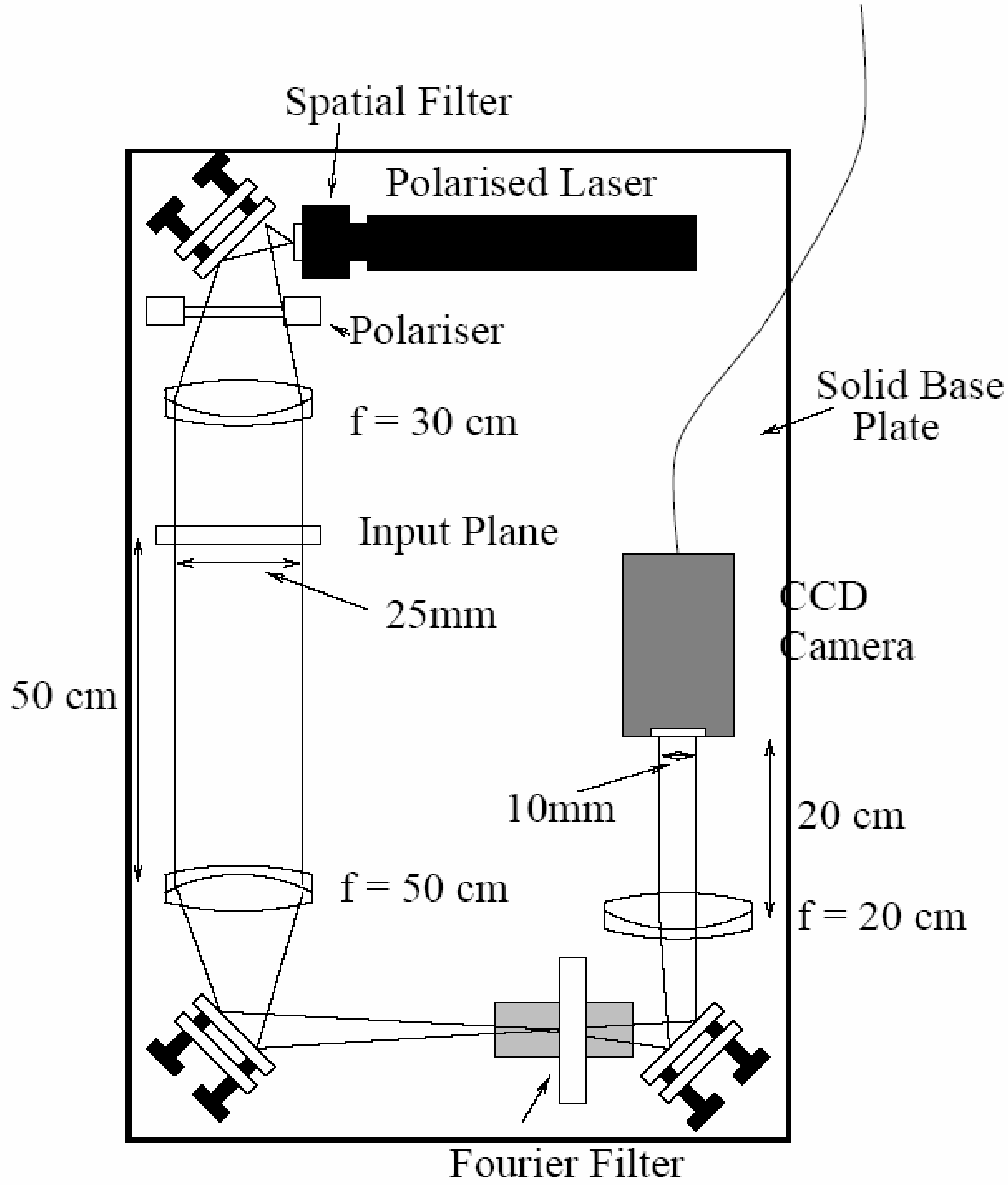
$$F(u,v) \quad u = \frac{x}{\lambda f} \quad v = \frac{y}{\lambda f}$$

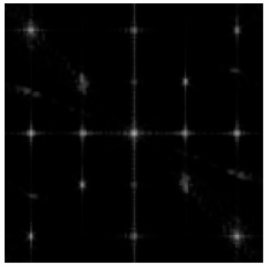
$$F(u,v)H(u,v)$$

$$u_4(x,y) = f_a(x,y) \odot h(x,y)$$

$$h(x,y) = \iint F(u,v)\exp(-\imath 2\pi(ux+vy))\mathrm{d}u\,\mathrm{d}v$$

$$g(x,y) = |f_a(x,y) \odot h(x,y)|^2$$


Spatial Filter
Polarised Laser
Polariser
Solid Base
Plate
f = 30 cm
Input Plane
25mm
CCD
Camera
50 cm
10mm
20 cm
f = 50 cm
f = 20 cm
Fourier Filter



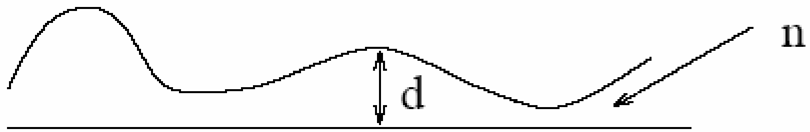
Fourier Transform

Original

Low pass filter

High pass filter

Phase Objects



$$f_a(x,y) = \exp\left(\imath\phi(x,y)\right)$$

$$\phi(x,y) = \frac{2\pi n d(x,y)}{\lambda}$$

$$g(x,y) = |f_a(x,y)|^2 = 1$$

Dark Field

$$
\begin{aligned}
H(u,v) &= 0 \quad u^2+v^2=0 \\
&= 1 \quad \text{else}
\end{aligned}
$$

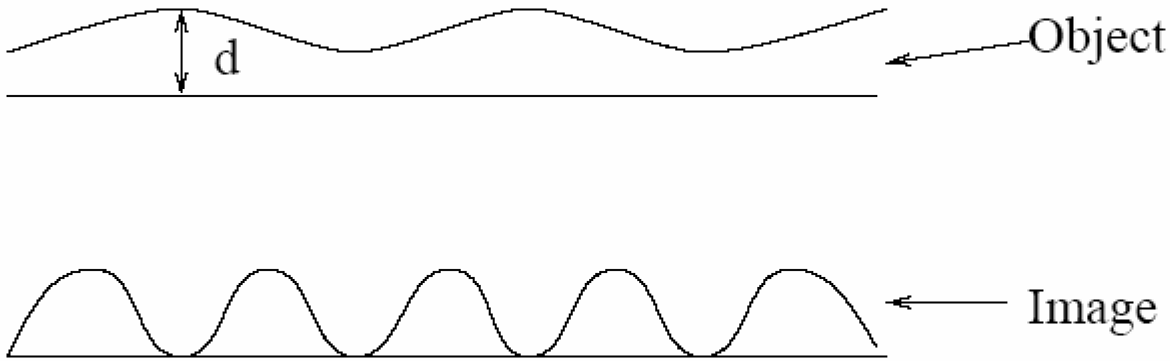
After filter

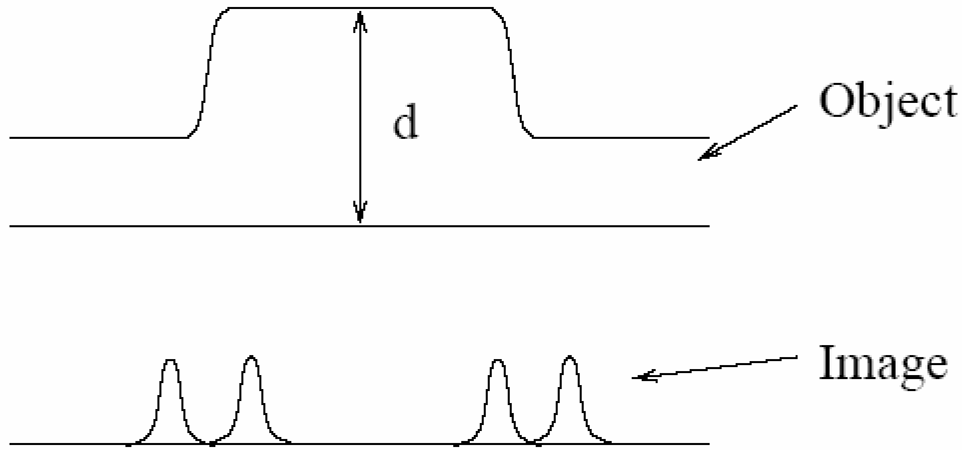
$$
\begin{aligned}
F(u,v)H(u,v) &= 0 \quad u^2+v^2=0 \\
&= \imath\Phi(u,v) \quad \text{else}
\end{aligned}
$$

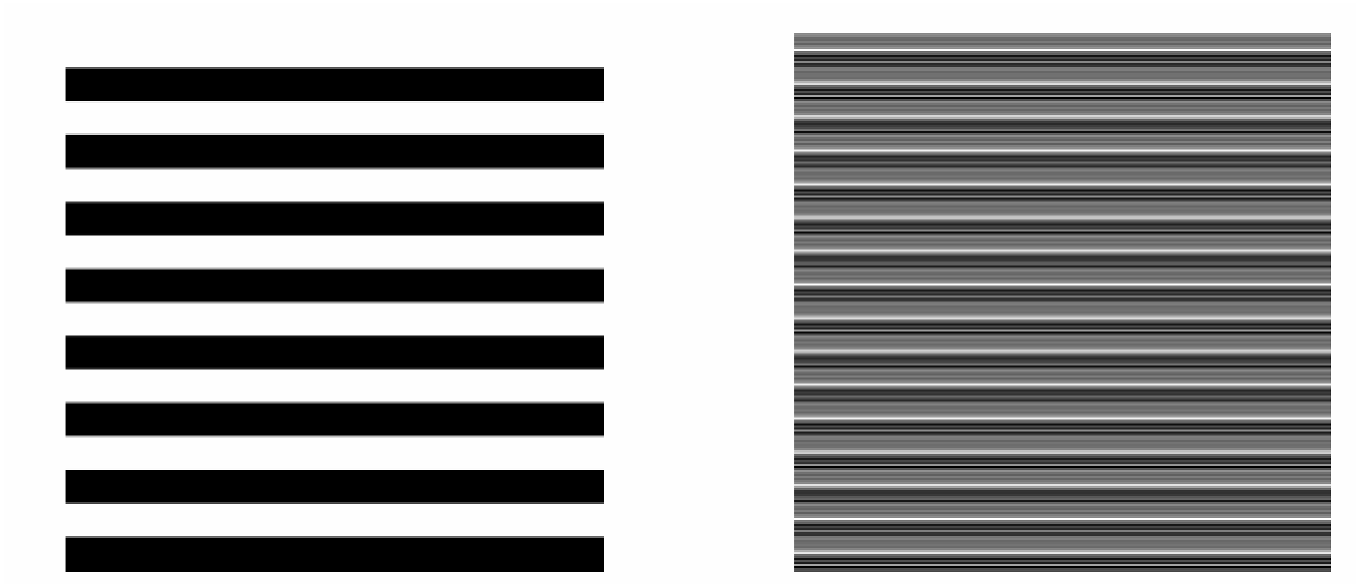
P_4 after second Fourier Transform

$$u_4(x,y) = \imath\phi(x,y)$$

$$g(x,y) = |\phi(x,y)|^2$$


d
Object
Image


d
Object
Image



Phase Contrast Filtering

Zernike 1940, (Nobel Prize 1953)

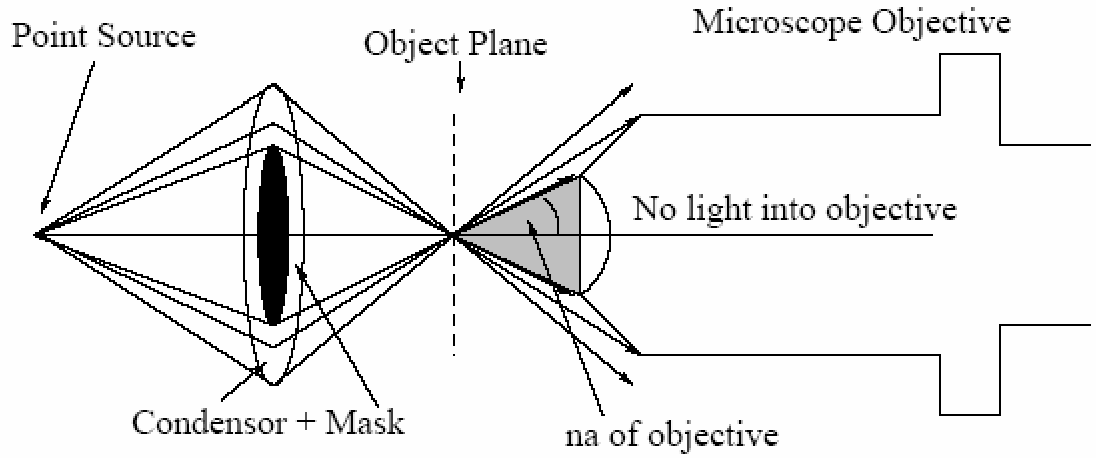
$$F(u,v) = \delta(u,v) + \imath\Phi(u,v)$$

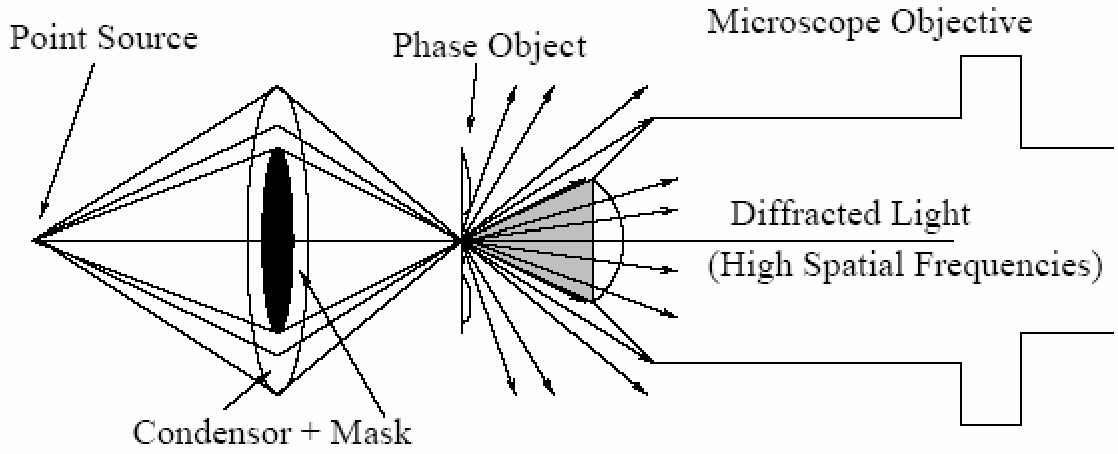
$$\begin{aligned} H(u,v) &= \exp(\imath\pi/2) \quad u^2+v^2=0 \\ &= 1 \quad \text{else} \end{aligned}$$

"dot" of $\lambda/4$ optical path length

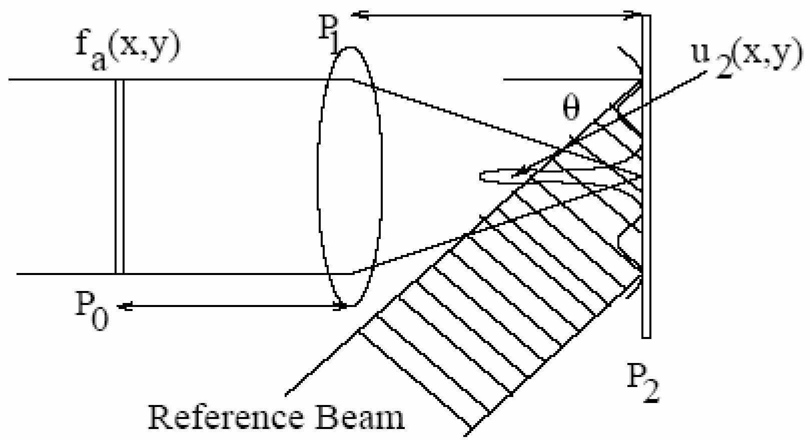
$$g(x,y) = |1+\phi(x,y)|^2 = 1 + 2\phi(x,y) + \phi^2(x,y)$$

$$g(x,y) \approx 1 + 2\phi(x,y)$$


Point Source
Object Plane
Microscope Objective
No light into objective
Condensor + Mask
na of objective


Point Source
Phase Object
Microscope Objective
Diffracted Light
(High Spatial Frequencies)
Condensor + Mask

Fourier Holograms



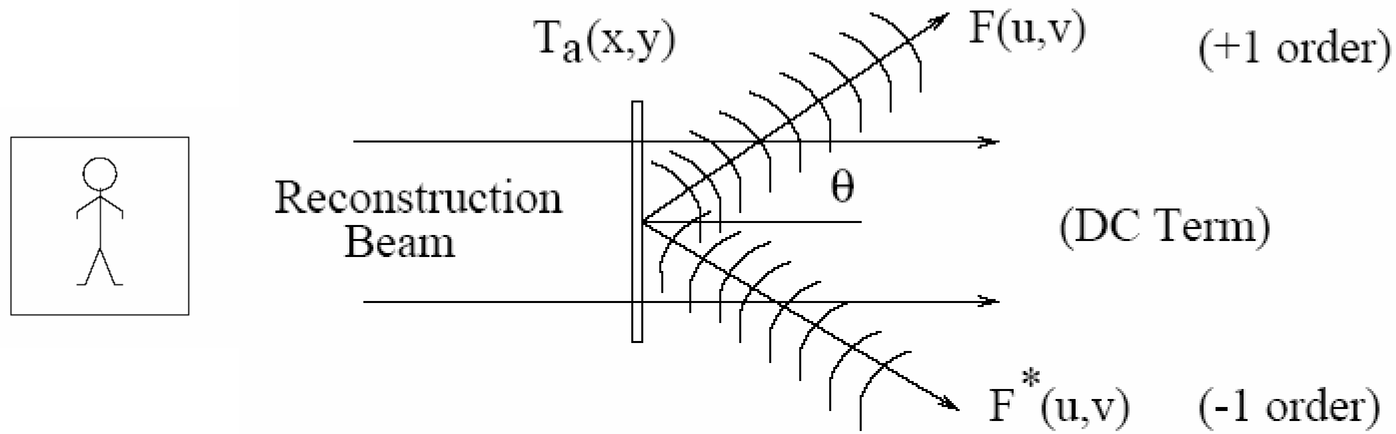
$$u_2(x,y) = F\left(\frac{x}{\lambda f}, \frac{y}{\lambda f}\right)$$

Intensity in P_2 is

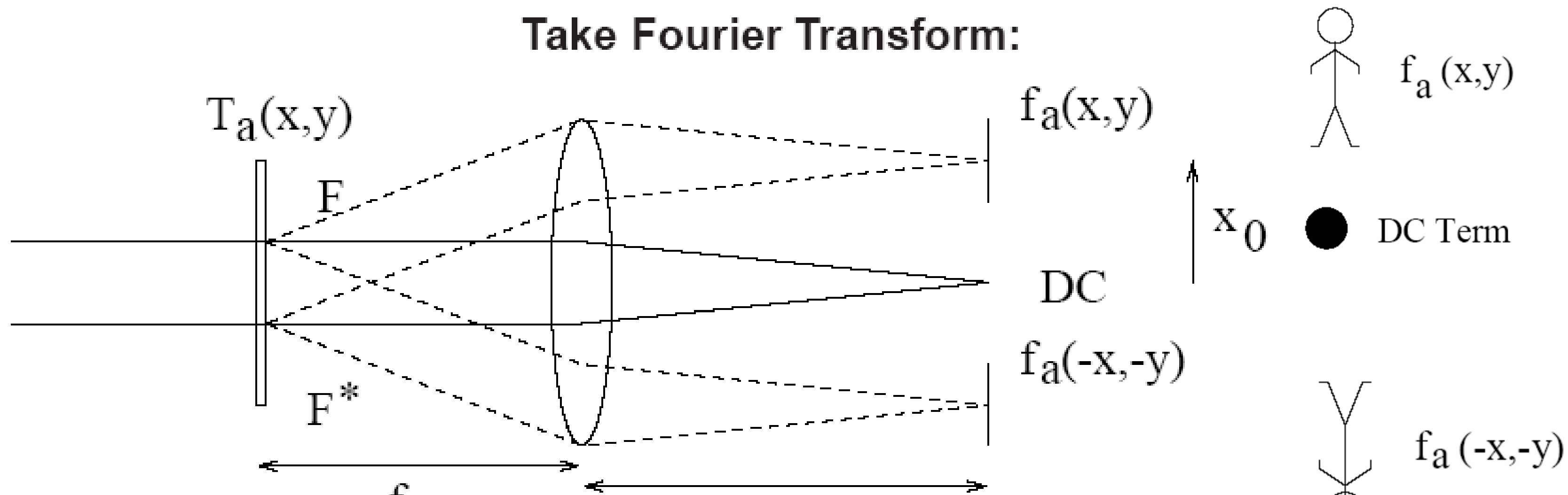
$$|r\exp(\imath\kappa x\sin\theta) + u_2(x,y)|^2$$
$$r^2 + |u_2(x,y)|^2 + 2r|u_2(x,y)|\cos(\kappa x\sin\theta - \Phi)$$

$$u_2(x,y) = |u_2(x,y)|\exp(\imath\Phi)$$

Reconstruction



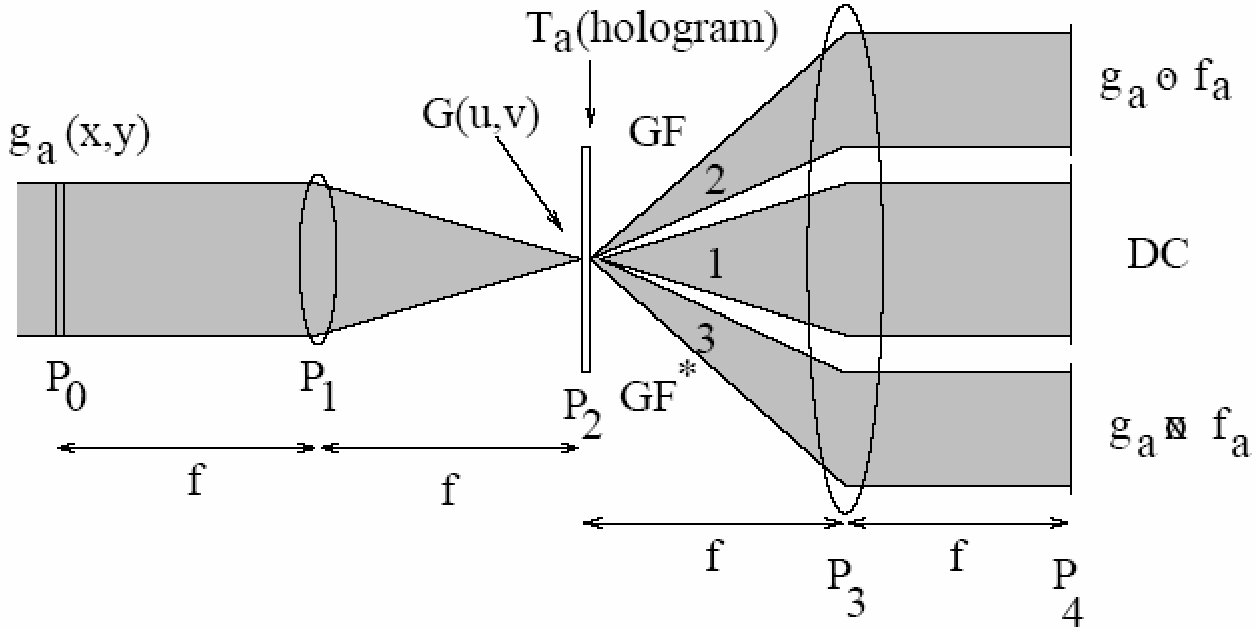
Take Fourier Transform:



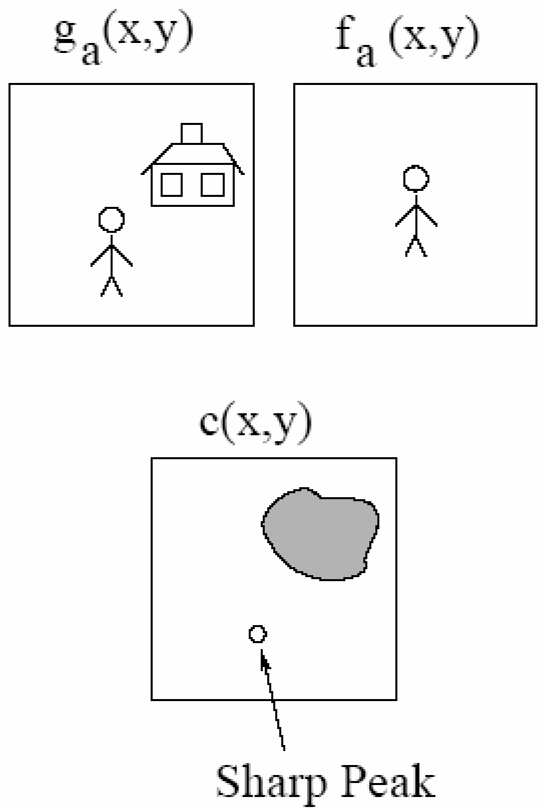
$$x_0 = f\tan\theta =\approx f\theta$$

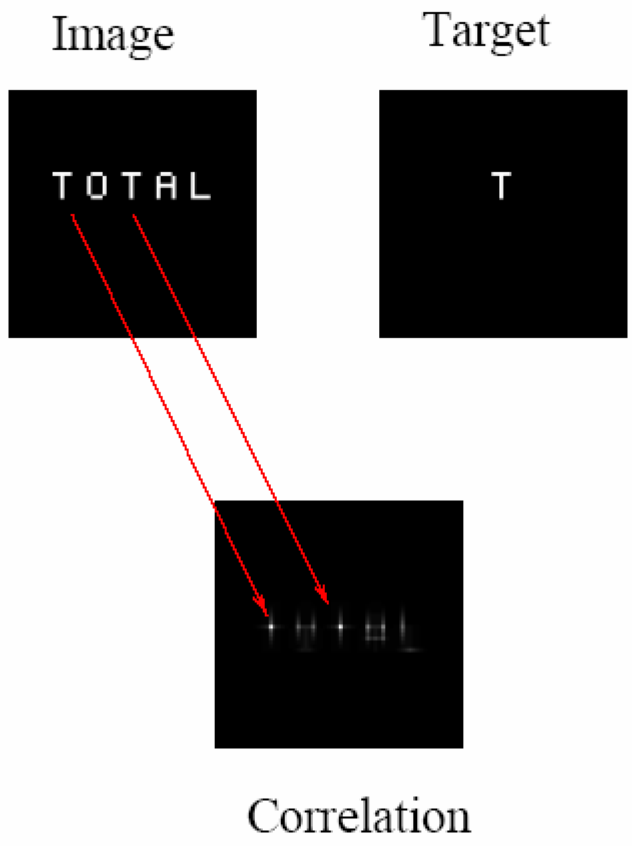
$$\delta(x,y) + f_a(x,y) \odot \delta(x - x_0) + f_a(-x,-y) \odot \delta(x + x_0)$$

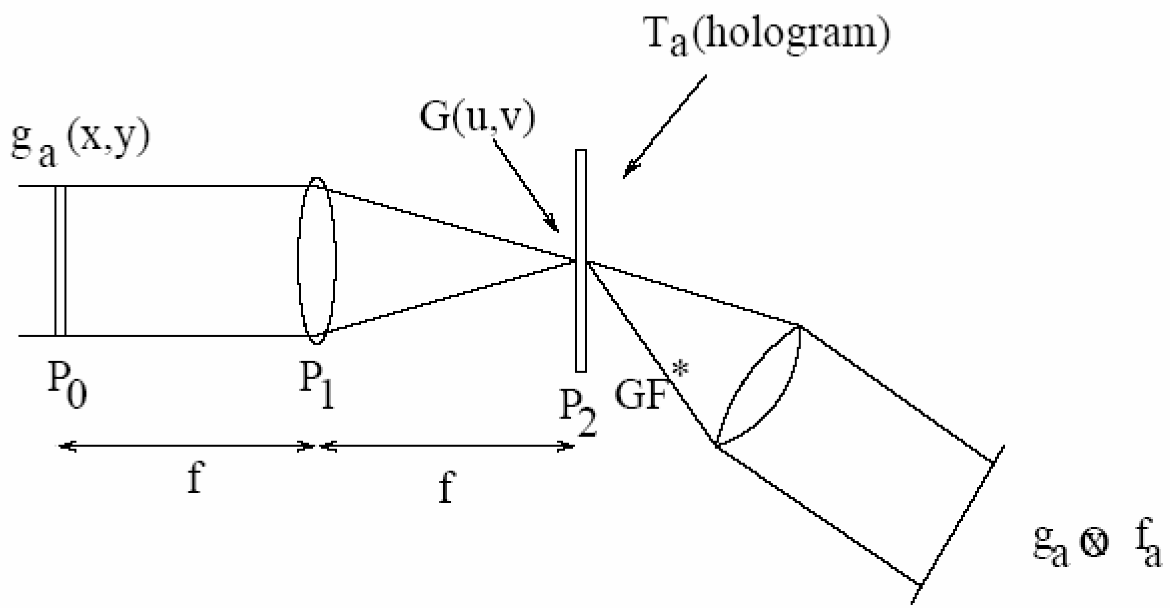
Vander Lugt Correlator (1966)



Target identification







T_a(hologram)
G(u,v)
g_a(x,y)
P_0
P_1
P_2
GF^*
f
f
$g_a \otimes f_a$